Exponents and Logarithm Notes

Contents:

Log Form

This is asking us what exponent (L) would we need acting on our base (b) to get to this number (N?)

$$\underbrace{\log_b N = L}_{\text{Logarithmic Form}} \rightleftarrows \underbrace{N = b^L}_{\text{Exponential Form}}$$

We can convert between logarithmic form and exponential form by taking the base of the log (b), bringing it to the other side of the equation, and making it the base of the exponent (L).

This is asking: what exponent (x) do we need acting on our base (2) to get to 8? We could also switch to exponential form to consider it further. Since 2*2*2 = 8, we know that x = 3.

$$\log_2 8 = x$$

$$2^x = 8$$

$$x = 3$$

This is asking: what number (x) raised to the power of 2 equals 16? We can solve this algebraically by converting to exponential form and then taking the square root of both sides. Remember that powers and roots are inverses of each other (cancel each other out).

$$\log_x 16 = 2$$

$$\sqrt{16} = \sqrt{x^2}$$

$$4 = x$$

This is asking: what is the answer (x) when 5 is raised to the power of 3? We convert to exponential form and then simplify.

$$\log_5 x = 3$$

$$x = 5^3$$

$$x = 125$$

If we don't see a log base, it's a 10. This is the only base that you don't have to write down for log.

$$\log x = 3$$
$$x = 10^3$$
$$x = 1{,}000$$

ln is also a log but with a base of e. Instead of writing $\log_e$ we can just write ln. It follows the same rules we've covered so far, so when we convert to exponential form, the base of the exponent is e.

$$\ln x = 7$$
$$x = e^7$$
$$x = 1{,}096.63\ldots\ldots$$

Log Properties

If you are adding things with like log bases, you can combine by multiplying the insides.

$$\log_b x + \log_b y \qquad \log_b(x \cdot y)$$

If you are subtracting things with like log bases, you can combine by dividing the insides.

$$\log_b x - \log_b y \qquad \log_b\left(\frac{x}{y}\right)$$

If you are taking the log of a number that is raised to an exponent, you can make the exponent into a coefficient by bringing it to the front of the log and multiplying.

$$\log_b x^f \qquad f \cdot \log_b x$$

We can use the properties of logs to expand.

$$\log_3 (x^4 y^7 z^8)$$

We are multiplying terms with the same base, so we can expand by adding.

$$\log_3 x^4 + \log_3 y^7 + \log_3 z^8$$

To expand the exponents, each exponent becomes the coefficient of the term it's in.

$$4 \log_3 x + 7 \log_3 y + 8 \log_3 z$$

We can use the properties of logs to simplify (condense) into one log.

$$6 \log_2 b - [4 \log_2 a + 3 \log c]$$

To condense the exponents, each coefficient becomes the exponent of the term it's in.

$$\log_2 b^6 - [\log_2 a^4 + \log_2 c^3]$$

In the parentheses, we are adding terms with the same log base, so we can condense (combine) by multiplying the insides.

$$\log_2 b^6 - [\log_2 (a^4 \cdot c^3)]$$

We are subtracting terms with the same log base, so we can combine by dividing.

$$\log_2 \left[\frac{b^6}{a^4 c^3}\right]$$

Log Equations

If we have an equation where both sides are just a log with the same base, we can drop the logs and just solve the equation that's left.

$$\log_3 5x = \log_3 (3x+6)$$

$$5x = \cancel{3x} + 6$$
$$-3x \quad -\cancel{3x}$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

If we have an equation with multiple logs on one side, we can use the properties of logs we learned above to condense into one log and solve.

Since we are subtracting terms with the same log base, we can combine by dividing the inside numbers.

$$\log_6 (x+1) - \log_6 x = \log_6 29$$

$$\log_6 \left(\frac{x+1}{x}\right) = \log_6 29$$

Since we have only one log on each side of the equation and they have the same log base, we can drop the logs and solve the inside numbers.

$$x \cdot \frac{x+1}{x} = 29 \cdot x$$

$$\cancel{x} + 1 = 29x$$
$$-\cancel{x} \qquad -1x$$

$$\frac{1}{28} = \frac{28x}{28}$$

$$\frac{1}{28} = x$$

In this example, we remember that ln is $\log_e$ and use our properties of logs to combine before solving.

$$\ln 2 - \ln(3x+2) = 1$$

Since we are subtracting terms with the same log base, we can combine by dividing the inside numbers.

$$\ln\left(\frac{2}{3x+2}\right) = 1$$

Since we know that ln is the same as $\log_e$ then we can convert to exponential form by make 1 the exponent of e. Algebraic explanation below.

$$\ln\left(\frac{2}{3x+2}\right) = 1$$

$$\log_e\left(\frac{2}{3x+2}\right) = 1$$

$$\frac{2}{3x+2} = e^1$$

$$(3x+2)\cdot\frac{2}{3x+2} = e^1\cdot(3x+2)$$

$$\frac{2}{e^1} = \frac{e^1(3x+2)}{e^1}$$

$$0.736 = 3x + \cancel{2}$$

$$-2 \qquad \cancel{-2}$$

$$\frac{-1.264}{3} = \frac{3x}{3}$$

$$-0.421 = x$$

Before we can convert from log form to exponential form, we have to isolate the log (have it be the only term on its side of the equal sign).

$$9\log_6(2a+1) + 6 = 33$$

$$-6 \qquad -6$$

To isolate the log, we need to use inverse operations to move the +6 and the *9 to the other side of the equation.

$$\frac{9\log_6(2a+1)}{9} = \frac{27}{9}$$

Now that our log is isolated, we can convert to exponential form.

$$\log_6(2a+1)=3$$

$$2a+1=6^3$$

$$2a+\cancel{1}=216$$
$$\cancel{-1}\quad -1$$

$$\frac{2a}{2}=\frac{215}{2}$$

$$a=107.5$$

In this example, we need to isolate the e first before solving for our x.

$$\frac{2e^{8x}}{2}=\frac{45}{2}$$

To isolate the e as its own base, we have to divide both sides by 2 (since that's the inverse of multiplying by 2).

$$e^{8x}=22.5$$

$$\ln(e^{8x})=\ln(22.5)$$

The inverse of e is ln, so we take the ln of both sides to cancel out the e.

$$\frac{8x}{8}=\frac{3.114}{8}$$

$$x=0.389\ldots$$

Using Like Bases to Solve Exponential Equations

$$16^{3x-4} = 64^{4x+3}$$

Since 16 and 64 can both be represented as 2 raised to a power, we can change 16 to 2^4 and 64 to 2^6.

$$(2^4)^{3x-4} = (2^6)^{4x+3}$$

Using our power rule of exponents, we multiply the exponents.

$(x^m)^n = x^{m*n}$

$$2^{12x-16} = 2^{24x+18}$$

Since we have two exponential expressions equal to each other that have the same base (2), we can drop the bases and solve the exponents.

$$12x - 16 = \cancel{24x} + 18$$
$$-24x \qquad \cancel{-24x}$$

$$-12x - \cancel{16} = 18$$
$$+\cancel{16} \qquad +16$$

$$\frac{-12x}{-12} = \frac{34}{-12}$$

$$x = -17/6$$

Change of Base Formula

Everything we've looked at so far assumes that the log bases are equal. If they are not the same, there is a formula we can use to move to a different base. You can use this to move to base 10 (m=10) if that's easier for you (or your calculator) to work with. You can also switch to a different log base if it will make it easier for you to solve a given problem.

$$\log_b N \rightarrow \frac{\log_m N}{\log_m b}$$

Change of Base Formula
The numerator is the log with a new base of the N. The denominator is the log with the same new base of the original log base (b)

$$\log_8 16 \rightarrow \frac{\log_2 16}{\log_2 8}$$

$$\frac{4}{3}$$

Since we know that both 8 and 16 can be solved with a base of 2, we used the change of base formula to switch to a base of 2 where we could then simplify the top and bottom of the fraction.